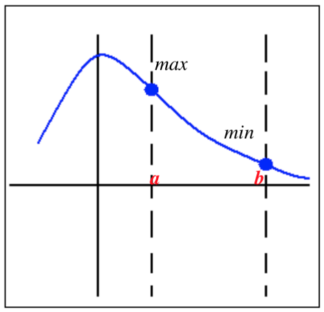
Extreme Value Thm WS

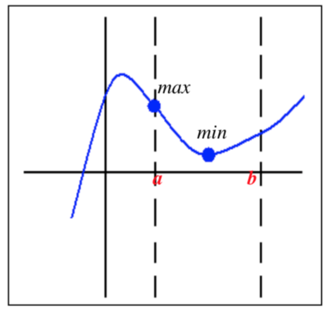
Name: Block: Seat:

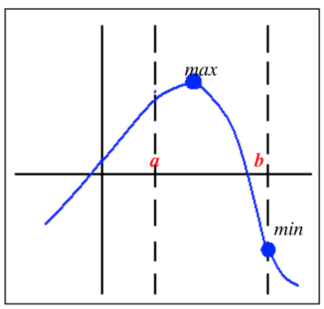
Definitions

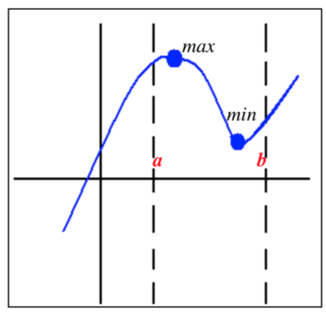
Absolute Extrema: refers to the maximum and minimum values of a function in a specified interval.

Extreme Value Theorem: (Max-Min Existence)
If f is **continuous** on a **closed** interval $[a, b]$, then f attains both a maximum and minimum value there.









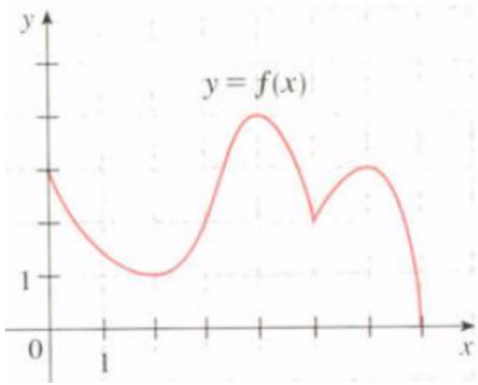
Procedure

1. Find the derivative of the function.
2. Find the critical numbers: Any value of x where $f'(x) = 0$ or DNE
3. Create a table of *candidates* containing:
   (a) The x values found in step 2, next to their corresponding $f(x)$ value.
   (b) The end points (the a and b value of $[a, b]$ and their $f(a)$ and $f(b)$ value)
   (c) NOTHING ELSE

x						
$f(x)$						

4. Identify the absolute maximum and minimum of the stated interval by comparing their $f(x)$-values. The maximum or minimum value can exist at the endpoints ONLY IF the stated interval is closed $[a, b]$.

Examples

1. 

   Use the graph of $f(x)$ to state the following

   (a) absolute maximum and absolute minimum value

   (b) local maximum and minimum value(s).

2. (a) Find the maximum/minimum values of the function $f(x) = x^3 - 3x^2 - 9x + 4$ in the interval $[-4, 2]$.

x	
$f(x)$	

   (b) Consider the open interval $(-4, 2)$. Would your results change?

3. (a) Find the maximum/minimum values of the function $f(x) = x^2 + \dfrac{2}{x}$ in the interval $[\frac{1}{2}, 2]$.

x	
$f(x)$	

   (b) Consider the open interval $(\frac{1}{2}, 2)$. Would your results change?

Practice

Determine the absolute maximum and absolute minimum value over the stated interval by applying the Extreme Value Theorem.

1. $f(x) = x^2 + 4x + 4$ on the interval $[-4, 0]$

2. $f(x) = x^2 + 3x$ on the interval $[-2, 1]$

3. $f(x) = x^3 - 3x + 1$ on the interval $\left(-\frac{3}{2}, 3\right)$

4. $f(x) = x^3 - 3x^2$ on the interval $[-1, 3]$

5. $f(x) = x^3 - 12x$ on the interval $(0, 4)$

6. $f(x) = \dfrac{x}{x-2}$ on the interval $[3, 5]$

7. $f(x) = \dfrac{1}{x}$ on the interval $[-1, 3]$

8. $f(x) = \dfrac{1}{1+x^2}$ on the interval $(-3, 3)$

9. $f(x) = \sqrt[3]{x}$ on the interval $[-1, 27]$

10. $f(x) = \sqrt{9 - x^2}$ on the interval $[-1, 2]$